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CONDENSATION COARSENING OF AEROSOL

PARTICLES IN A COOLING VAPOR—GAS FLOW

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A system of equations is obtained to describe the motion of a vapor—gas flow with aerosol particles in a channel of variable cross-sectional area in the presence of external heat transfer. The system is solved analytically for a quasiequilibrium process.

The principle of the condensation-associated coarsening of highly dispersed aerosol particles during the cooling of a vapor—gas flow is used to increase the efficiency of several existing wet-type dust catchers and separators [1-5] and to develop new designs [6-11]. However, broad use of the method is being held up by insufficient research on the process, which makes it difficult to develop simple, effective, and economical equipment for the fine cleaning of gases.

The goal of the present study is to obtain a model of the process of condensation coarsening of aerosol particles in a cooling vapor—gas flow with a variable cross section along its length.

The cooling of a vapor—gas mixture moving along a colder surface is accompanied by heat transfer through the gaseous boundary layer adjacent to it and subsequent condensation. If the mixture contains a disperse phase (solid or liquid aerosol particles), then condensation occurs not only on the surface of the channel but also on the particles. Also, in the event of significant supersaturation, nuclei are also formed — small drops of liquid (homogeneous condensation). Considering the small diameter of the nuclei (about 10^{-9} m [12]) and assuming that supersaturation is negligible (significant supersaturation being possible only in a pure gas or at very high cooling rates) and that the concentration of the disperse phase is relatively high, it can be reasoned that the content of liquid in nuclei is negligibly small and that the process of their formation need not be examined in the present case.

The relationship between the masses of condensate formed on the particles and on the channel surface undergoing cooling depends on the amount of supersaturation and the concentration of the disperse phase in the mixture. Amelin proved theoretically and it was confirmed empirically [12] that in the case of a large number of condensation nuclei (aerosol particles) in a flow, considerably more vapor condenses on them than on the walls. Thus, with a numerical concentration of particles of 10^8 m⁻³, 99% of all condensate is formed on them [12]. Condensation on the walls may be substantial only in the case of their film spraying or when they have a developed surface.

Condensation growth of aerosol particles can be speed up by increasing the partial vapor pressure by expanding the vapor—gas flow in various types of diffusers as a result of a reduction in its velocity. Landau and Lifshitz showed [13] that vapor condensation is

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impossible with an increase in pressure in a homogeneous vapor—gas mixture because the increase in partial pressure is accompanied by an increase in the temperature of the system, and its state deviates from equilibrium in the direction of a decrease in relative humidity. However, with the flow of a two-phase mixture — which contains a liquid phase as well as the vapor phase — condensation is possible if the heat given off here is insufficient to heat the condensed part of the system to a new equilibrium temperature corresponding to a new pressure (such as with a relatively high content of liquid phase or a slight increase in pressure). In this case, some of the vapor condenses, liberating a deficient quantity of heat.

To calculate the process of condensation coarsening of aerosol particles in a vapor—gas flow, we will examine the unidimensional motion of a two-phase vapor—gas mixture in a channel with a longitudinally varying cross-sectional area in the presence of external heat transfer.

The flow will be assumed to be steady. We also make the following assumptions: 1) the particles are drops of condensate; the mechanism remains the same in the case of solid particles or drops of a different liquid [12], but acceleration of the process may be achieved by contraction of the flow in a convergent duct rather than expansion in a diffuser; it was shown in [12] that with a sufficiently long time and high intensity of the process, condensate drop size depends little on the initial size of the condensation nuclei; 3) there is no interaction between particles and their form does not deviate from spherical; the validity of this assumption was proven in [15] for droplets no less than 15–20 μm in diameter at flow velocities up to 100 m/sec and numerical concentrations of the disperse phase no greater than 10^{16} m^{-3} ; 4) the volume occupied by the particles is small compared to the volume of the vapor—gas mixture; in the case of highly dispersed particles this condition is satisfied even with significant numerical concentrations; 5) the gas and vapor are calorimetrically perfect and are in dynamic and thermal equilibrium with each other, forming a homogeneous uniform medium.

The basic equations of the model of the process are the laws of conservation of mass, energy, and momentum.

We write the equations of conservation of mass of the gas (continuity) and the drop—vapor mixture as follows:

$$\frac{1}{F} \frac{dF}{dz} + \frac{1}{w} \frac{dw}{dz} + \frac{1}{\rho_g} \frac{d\rho_g}{dz} = 0, \quad (1)$$

$$\frac{1}{F} \frac{dF}{dz} (\rho_{\pi} w + nmu) + \frac{d}{dz} (\rho w + nmu) = 0. \quad (2)$$

It follows from the third assumption that in the absence of the settling of particles on the wall, the number of particles passing per unit time through any cross section of the channel remains constant, and we can write the equation of conservation of their numerical rate of flow

$$\frac{1}{F} \frac{dF}{dz} + \frac{1}{u} \frac{du}{dz} + \frac{1}{n} \frac{dn}{dz} = 0. \quad (3)$$

Since the flow is steady and since external forces are absent, the sum of the total pressure of the gas phase and the momentum of the disperse phase remains a constant value. Then the law of conservation of momentum can be represented as follows:

$$\frac{d}{dz} [(\rho_g + \rho_v) w^2 + nmu^2] + \frac{dP}{dz} = 0. \quad (4)$$

We write the equation of energy conservation in the presence of external heat transfer through the channel wall:

$$\begin{aligned} & \frac{1}{F} \frac{dF}{dz} \left[\rho_g w \left(i_g + \frac{w^2}{2} \right) + \rho_v w \left(i_v + \frac{w^2}{2} \right) + nmu \left(i_c + \frac{u^2}{2} \right) \right] + \\ & + \frac{d}{dz} \left[\rho_g w \left(i_g + \frac{w^2}{2} \right) + \rho_v w \left(i_v + \frac{w^2}{2} \right) + nmu \left(i_c + \frac{u^2}{2} \right) \right] + \frac{1}{F} \frac{dQ}{dz} = 0. \end{aligned} \quad (5)$$

Assuming that the vapor—gas mixture obeys the ideal gas laws, we write the following equation of state for it:

$$\frac{1}{T_g} \frac{dT_g}{dz} \left(\frac{\rho_g}{M_g} + \frac{\rho_v}{M_v} \right) + \frac{1}{M_g} \frac{d\rho_g}{dz} + \frac{1}{M_v} \frac{d\rho_v}{dz} - \frac{1}{RT_g} \frac{dP}{dz} = 0. \quad (6)$$

To allow for the exchange of mass, energy, and momentum between phases, the above conservation and state equations must be supplemented by the three corresponding relations. We write the equations describing heat and mass transfer between the phases in a form similar to that in [16]:

$$m \frac{di_k}{dz} = r \frac{dm}{dz} + \frac{1}{u} \alpha \pi d^2 (T_g - T_c), \quad (7)$$

$$\frac{dm}{dz} = - \frac{\beta \pi d^2 (p_c - p_v)}{B_v T_c}. \quad (8)$$

To account for the exchange of momentum between phases, we use the equation of motion of a single drop:

$$u \frac{du}{dz} = \frac{3}{4} \frac{\psi}{d} \frac{\rho_g}{\rho_c} |w - u|(w - u). \quad (9)$$

We express the temperature dependence of the saturation vapor pressure above the drops, which enters into (8), by means of a formula obtained from the Clausius—Clapeyron equation [17], introducing into it the Kelvin correction for the radius of curvature of the drop:

$$p_c = \exp \left[\frac{1}{B_v T_c} \left(\frac{4\sigma}{\rho_c d} - r \right) + A \right]. \quad (10)$$

System (1)–(10), with prescribed parameters of the two-phase flow at the channel inlet and known functions $F = f(z)$ and $Q = f(z)$, can be solved only by numerical methods. It is also quite difficult to determine the heat- and mass-transfer coefficients, since the experimental data available on this subject is quite contradictory.

In the case of highly dispersed particles less than 10 μm in diameter and flow velocities up to 100 m/sec, the system of equations obtained can be simplified considerably and reduced to a form permitting an analytic solution. As was shown in [15], the deviation of particle velocity in this case from the gas velocity is no greater than 10%. Thus, it can be assumed that $u = w$, i.e., there is mechanical equilibrium between the phases in the flow.

The time of temperature equalization between droplets and gas is very short and is usually much shorter than the time it takes for the process to occur [12]. The study [17] presented results of theoretical and experimental investigations conducted by several authors which suggest that even with supersonic flow velocities, the condensation process is usually close to equilibrium. Here, the vapor pressure is equal to the saturation vapor pressure above the drops and the temperatures of the phases are equal in any cross section of the channel. It can therefore be assumed that in each cross section condensation occurs in exactly the same manner as in a medium with constant parameters equal to the values of the parameters in this section, $p_v = p_c$ and $T_c = T_g = T$. A similar assumption was made in [18].

The critical supersaturation $S_{cr} = p_c/p_s$ for drops 0.1 μm in diameter is on the order of 1.01 [12]. Thus, it can be assumed with a high degree of accuracy that $S_{cr} = 1$ for particles more than 0.1 μm in diameter.

The last assumptions significantly simplify calculation of the process of condensation coarsening of aerosol particles in a vapor—gas flow and have little effect on its accuracy. Thus, the assumptions made regarding the equilibrium of the process do not introduce a significant error relative to the exact solution even in the case of substantial accelerations of the flow, such as in shocks [19].

To calculate a process which is quasiequilibrium with respect to rates and temperatures, instead of heat- and mass-transfer equations (7) and (8) it is sufficient to use a formula obtained from the Clausius—Clapeyron equation [17]. Then, with allowance for the additional assumptions, the resulting system of equations can be written in the form

$$\frac{1}{F} \frac{dF}{dz} + \frac{1}{w} \frac{dw}{dz} + \frac{1}{\rho_g} \frac{d\rho_g}{dz} = 0, \quad (11)$$

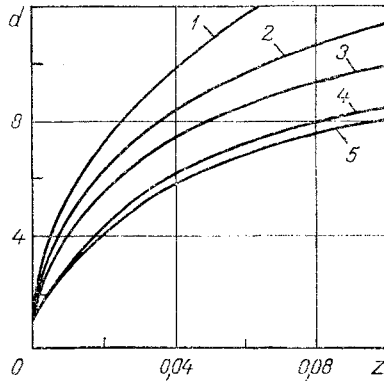


Fig. 1.

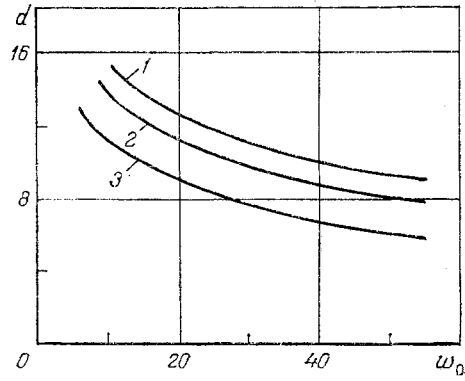


Fig. 2

Fig. 1. Change in diameter of aerosol particles d (μm) along the channel z (m) ($D = 30$ mm, $T_0 = 298^\circ\text{K}$, $d_0 = 1 \mu\text{m}$, $n_0 = 10^{10} \text{ m}^{-3}$): 1) $w_0 = 10$ m/sec, $q = 10$ kW/m 2 ; 2) 30 and 15; 3) 30 and 10; 4) 50 and 10; 5) 30 and 5.

Fig. 2. Dependence of the diameter of the aerosol particles d (μm) on the velocity of the vapor-gas flow w_0 (m/sec) ($D = 30$ mm, $T_0 = 298^\circ\text{K}$, $d_0 = 1 \mu\text{m}$, $n_0 = 10^{10} \text{ m}^{-3}$, $z = 0.1$ m): 1) $q = 15$ kW/m 2 ; 2) 10; 3) 5.

$$\left(\frac{1}{F} \frac{dF}{dz} + \frac{1}{w} \frac{dw}{dz} \right) (\rho_v + nm) + \frac{d\rho_v}{dz} + m \frac{dn}{dz} + n \frac{dm}{dz} = 0, \quad (12)$$

$$\frac{1}{F} \frac{dF}{dz} + \frac{1}{w} \frac{dw}{dz} + \frac{1}{n} \frac{dn}{dz} = 0, \quad (13)$$

$$w^2 \left(\frac{d\rho_g}{dz} + \frac{d\rho_v}{dz} + m \frac{dn}{dz} + n \frac{dm}{dz} \right) + 2w \frac{dw}{dz} (\rho_g + \rho_v + nm) + \frac{dP}{dz} = 0, \quad (14)$$

$$\begin{aligned} & \left(\frac{1}{F} \frac{dF}{dz} + \frac{1}{w} \frac{dw}{dz} \right) \left[\rho_g \left(i_g + \frac{w^2}{2} \right) + \rho_v \left(i_v + \frac{w^2}{2} \right) + \right. \\ & \left. + nm \left(i_c + \frac{w^2}{2} \right) \right] + \frac{d}{dz} \left[\rho_g \left(i_g + \frac{w^2}{2} \right) + \rho_v \left(i_v + \frac{w^2}{2} \right) + nm \left(i_c + \frac{w^2}{2} \right) \right] + \frac{1}{wF} \frac{dQ}{dz} = 0, \quad (15) \end{aligned}$$

$$\frac{1}{T} \frac{dT}{dz} \left(\frac{\rho_g}{M_g} + \frac{\rho_v}{M_v} \right) + \frac{1}{M_g} \frac{d\rho_g}{dz} + \frac{1}{M_v} \frac{d\rho_v}{dz} - \frac{1}{RT} \frac{dP}{dz} = 0, \quad (16)$$

$$p_v = \exp \left(A - \frac{r}{B_v T} \right). \quad (17)$$

System (11)-(17) can be solved analytically with known values of all of the parameters of the two-phase flow at the channel inlet (at $z = 0$) and prescribed functions $F = f(z)$ and $Q = f(z)$. Its solution relative to the particle diameter d will be an expression of the following form:

$$d^3 = d_{\max}^3 - \frac{1}{\pi N \rho_c} \frac{a - bT - 2Q}{2r - B_v T \left[1 - c \exp \left(\frac{r}{B_v T} - A \right) \right]}, \quad (18)$$

in which the value of the maximum particle diameter d_{\max} which is possible with the given parameters is found from the condition that all of the vapor in the system condenses on the particles:

$$d_{\max}^3 = \frac{6}{\pi \rho_c} \frac{G_{v0} + G_{c0}}{N}, \quad (19)$$

while the temperature in the flow is found from solution of the transcendental equation

$$G \left[\frac{1}{F \left(\frac{2r}{B_v T} - 1 \right) \exp \left(A - \frac{r}{B_v T} \right) + c} \right]^2 + \frac{a - bT - 2Q}{\frac{2r}{B_v T} \exp \left(\frac{r}{B_v T} - A \right) - 1} + G_g B_g T = 0. \quad (20)$$

In Eqs. (18)-(20), the values of a , b , c , G , and N are found from the flow parameters at the channel inlet:

$$\frac{a}{2} = G_g \left(i_g + \frac{\omega_0^2}{2} \right) + G_v \left(i_v + \frac{\omega_0^2}{2} \right) + G_c \left(i_c + \frac{\omega_0^2}{2} \right), \quad (21)$$

$$b = G_g (2c_g + B_g) + 2(G_v + G_c) c_c, \quad (22)$$

$$c = \omega_0^2 (\rho_{g0} + \rho_{v0} + n_0 m_0) + P_0, \quad (23)$$

$$G = G_g + G_v + G_c = \omega_0 F_0 (\rho_{g0} + \rho_{v0} + n_0 m_0), \quad (24)$$

$$N = n_0 \omega_0 F_0. \quad (25)$$

Equations (18)-(25) make it possible to find the diameter of aerosol particles in any channel cross section. Figure 1 shows some results of the calculations.

The proposed model was verified empirically in a cylindrical channel 30 mm in diameter and 200 mm in length equipped with a cooling hose. The model was verified on an air-water vapor-drop system. The main parameters were varied within the following ranges: $T_0 = 293-363^\circ\text{K}$, $w_0 = 5-50$ m/sec, $d_0 = 1-10$ μm , $n_0 = 10^8-10^{12}$ m^{-3} , $q = 5-15$ kW/m^2 . The disperse composition of the aerosol particles in the vapor-gas mixture was determined with an AZ-5M aerosol particle counter.

The tests showed that the proposed model qualitatively accurately describes the process being examined. However, the large error of the determination of particle size in the mixture by means of the AZ-5M counter (up to 20% for particles larger than 1 μm in diameter), as in the case of other existing instruments [20], prohibits a quantitative comparison of the theoretical and experimental data. Thus, the proposed model was checked in accordance with the amount of condensate formed on the particles during their passage through a cylindrical channel, which can easily be calculated from Eqs. (18)-(25). The condensate obtained in the channel was separated from the vapor-gas mixture by a centrifugal separator with a screw swirler and a model FPP-15-1.7 Petryanov filter [21]. The discrepancy between the experimental and theoretical data in the above ranges of the basic parameters was no greater than 30%.

The calculations and experiments show that condensation coarsening makes it possible to significantly increase the size of the particles (from 1 to 10-15 μm) and to thereby ensure their efficient separation. The final size of the particles is determined mainly by the surface heat flux, and at gas velocities greater than 30 m/sec and numerical particle concentrations greater than 10^{12} m^{-3} it depends slightly on flow velocity (Fig. 2) and initial particle size. This is because in this case the volume of the condensate formed on each particle is much greater than the volume of the particle itself. Condensation coarsening of aerosol particles in a vapor-gas flow with external heat transfer is most expediently accomplished at low concentrations of the disperse phase (to 10^{12} m^{-3}), high thermal loads, and low flow velocities (to 30 m/sec). At higher concentrations, the small volume of the liquid phase condensed on each particle cannot lead to its appreciable coarsening. An increase in flow velocity leads to an increase in energy expenditure on condensation with a negligible increase in the final size of the aerosol particles.

The above model was used to calculate the efficiency of the removal of aerosol particles in a condensation-type centrifugal separator [8]. Completed experimental studies also confirmed the good agreement between the empirical and theoretical data [15].

NOTATION

A , coefficient in Eq. (10), dependent on the properties of the condensing substance [17]; B_g , B_v , specific gas constants for the gas and vapor, J/(kg·K); c_g , c_c , specific heats of the gas and condensate, J/(kg·K); D , channel diameter, m; d , diameter of aerosol particle, m; d_{max} , maximum diameter of aerosol particle, m; F , cross-sectional area of channel, m^2 ; G , mass velocity of the two-phase vapor-gas mixture, kg/sec; G_g , G_v , G_c , mass velocities of the gas, vapor, and condensate, kg/sec; i_g , i_v , i_c , enthalpies of the gas, vapor, and

condensate, J/kg; M_g , M_v , molar masses of the gas and vapor, kg/kmole; m , mass of aerosol particle, kg; N , numerical flow rate of aerosol particles, sec^{-1} ; n , numerical concentration of aerosol particles in the gas, m^{-3} ; P , pressure of vapor—gas mixture, Pa; p_s , p_c , pressures of saturated vapor above the plane surface and above the drops, Pa; p_g , p_v , partial pressures of gas and vapor, Pa; Q , heat flow through channel wall, W; q , surface heat flux through channel wall, W/m^2 ; $R = 8.31 \cdot 10^3$ J/(kmole·K), universal gas constant; r , heat of phase transformation, J/kg; $S_{cr} = p_c/p_s$, critical supersaturation; T , equilibrium temperature, K; T_g , T_c , temperatures of vapor—gas mixture and drop of condensate, K; u , velocity of aerosol particles, m/sec; w , velocity of vapor—gas flow, m/sec; z , longitudinal coordinate, m; α , heat-transfer coefficient, $\text{W}/(\text{m}^2 \cdot \text{K})$; β , heat-transfer coefficient, m/sec; ρ_g , ρ_v , ρ_c , densities of vapor, gas, and condensate, kg/m^3 ; σ , surface tension of condensing liquid, N/m; ψ , drag coefficient. Subscripts: 0, parameters at the channel inlet.

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